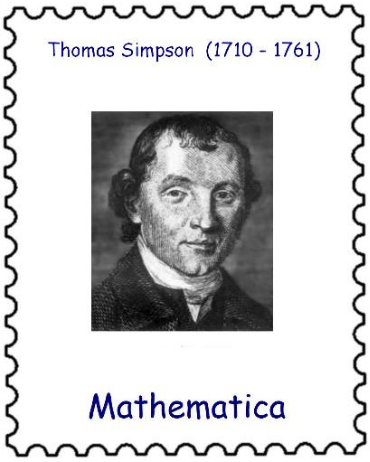
THOMAS SIMPSON (August 20, 1710 – May 14, 1761)

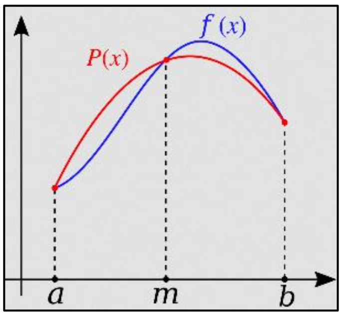
by HEINZ KLAUS STRICK, Germany



Two rules of integral calculus remind us of the English mathematician THOMAS SIMPSON:

In the so-called $\frac{1}{3}$ *rule*, the function values at the beginning, in the middle and at the end of the integration interval are used to approximate the graph of the function *f* under consideration by a quadratic parabola *P* and thus determine the content of the area between graph and *x*-axis approximately:

$$\int_a^b f(x)dx \approx \tfrac{1}{3} \cdot h \cdot \left[f(a) + 4 \cdot f(\tfrac{a+b}{2}) + f(b) \right],$$



where $h = \frac{b-a}{2}$ is equal to the respective distance of the three supporting points, cf. *Wikipedia* figure on the right by POPLETIBUS.

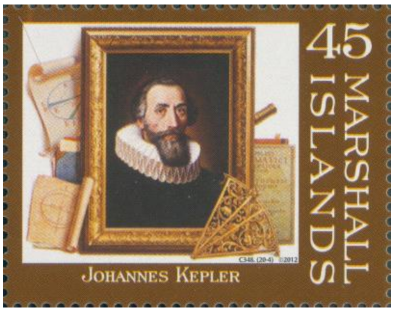
In SIMPSON's so-called $\frac{3}{8}$ *rule*, four grid points are considered and an approximation by a cubic curve is examined accordingly.

Then we get:

$$\int_a^b f(x)dx \approx \tfrac{3}{8} \cdot h \cdot \left[f(a) + 3 \cdot f(\tfrac{2a+b}{3}) + 3 \cdot f(\tfrac{a+2b}{3}) + f(b) \right],$$

where $h = \frac{b-a}{3}$ is equal to the respective distance between the four supporting points.

This approach can be generalised.



The idea of the $\frac{1}{3}$ *rule* also underlies the so-called *barrel rule* of JOHANNES KEPLER (1571-1630) – to determinate approximately the volume of a barrel by the rotation of a parabola.

THOMAS SIMPSON himself stated that he found the formula in ISAAC NEWTON, i.e. the rule should really bear NEWTON's name.

On the other hand, THOMAS SIMPSON was the first to publish the recursion equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

usually called *NEWTON's formula* for the approximate determination of the zero of a differentiable function *f* (in 1740, 13 years after NEWTON's death).

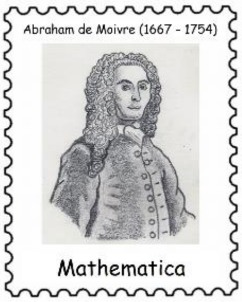
Thomas Simpson was born the son of a weaver in Market Bosworth, Leicestershire. He attended school there, but probably learnt little more than reading, writing and simple arithmetic.

At first he worked – like his father – as a weaver, until at the age of 14 he got hold of the arithmetic book by Edward Cocker, which had over 100 editions after its publication in 1678. He worked through the book on his own – obviously successfully, because in the following year the 15-year-old was able to leave his parental home to work as a mathematics teacher in the neighbouring town of Nuneaton (Warwickshire).

At the age of 19, Simpson married a widow whose son from his first marriage was older than the new husband. Two children were later born in this marriage.

In the meantime Simpson had also worked through works on astrology and he now earned part of his income as a fortune teller – until one day he went over the top and made the devil appear during a session ...

After this "scandal" he could no longer stay in Nuneaton. He fled to Derby, where he again worked as a weaver, and he gave arithmetic lessons at a night school.



In 1736 he moved to London, where he joined the *Spitalfields Mathematical Society*, an association of interested craftsmen, most of them weavers.

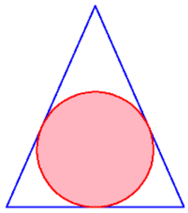
Simpson earned his living as a mathematics teacher at so-called *Penny universities*, which were coffee houses where one could listen to lectures, including on legal and artistic subjects, for the payment of an entrance fee of one penny. Abraham de Moivre also had to earn his living in this way, since as a French emigrant – despite Newton's support – he was not able to find a permanent job.

In the same year, Simpson published his first articles in *The Ladies Diary* (*Woman's Almanack - designed for the custom and diversion of the fair sex*), a much-read annual calendar with puzzles of all kinds, interesting news from science and mathematical problems. One of the articles dealt with *fluxions*, a subject which – because of the vociferous criticism of Bishop George Berkeley – had aroused great interest among women readers.

Simpson proved that he had understood Newton's differential calculus. In 1737, the ambitious work *Treatise of Fluxions* was published, which contained, among other things, Simpson's rule and Newton's formula, as well as 22 application problems in which he determined maxima and minima with the help of the zeros of the derivative, for example:

We are looking for …

- among all isosceles triangles whose sides touch a given circle, the triangle with the smallest area.
- among all right-angled triangles with a fixed hypotenuse, the triangle with the largest area,



- among all cones with the same surface area, the smallest in volume,
- the minimum distance of two bodies each moving at constant speed in a straight line from two places,
- the direction a boat should take to come as close as possible to a passing ship.

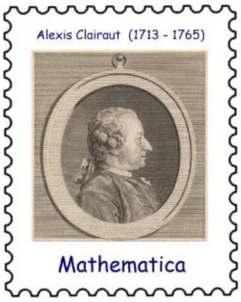
In 1740 followed *The Nature and Laws of Chance*, and in 1742 *The Doctrine of Annuities and Reversions* on probability and the calculation of insurance premiums. In the preface, Simpson praises de Moivre's *Doctrine of Chances*, published in 1738 in the 2nd edition, as an excellent book. De Moivre, however, complains about the numerous similarities with his book, but above all that he, who is dependent on the sale of his book, has a heavy loss of income.

Further books followed: *A Treatise of Algebra* (1745, with a total of 16 editions, also in other languages), *Elements of Plane Geometry* (1747, conceived as a textbook), *Trigonometry - Plane and Spherical* (1748, several editions, also in French).

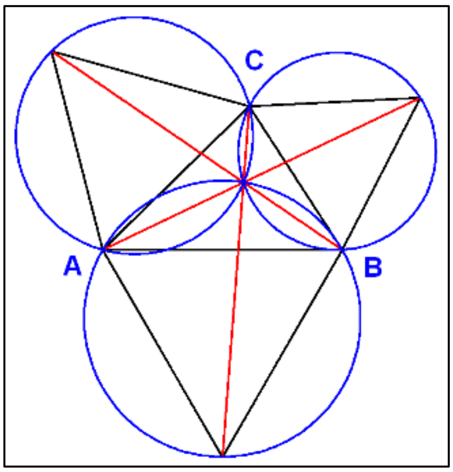
In the meantime Simpson was appointed head of the mathematics department of the *Royal Military Academy* in Woolwich (1743), an institution for future officers of the *Royal Artillery* and the *Royal Engineers*. From this time on, Simpson also became more involved in solving technical problems, such as the construction of fortifications and a bridge over the Thames.

In 1745 Simpson became a member of the *Royal Society*, and in 1758 also a member of the *Swedish Academy of Sciences*. From 1754 onwards, he was editor of *The Ladies Diary*, an activity that was very time-consuming due to the extensive correspondence.

In the following years, he wrote further books: *Doctrine and Application of Fluxions* (1750, an extended edition in two volumes), *Selected Exercises in Mathematics* (1752), *Miscellaneous Tracts on Some Curious Subjects in Mechanics* as well as *Physical Astronomy* and *Speculative Mathematics* (1757). The last work contains, among other things, an investigation of the moon's orbit. To determine the furthest point from the earth (*apogee*), he set up a differential equation – at the same time as Alexis Clairaut and independently of the latter – and solved it in general.



The book also contains a solution to a problem that Pierre de Fermat posed to Evangelista Torricelli in 1646: The search is for a point *D* whose sum of the distances of three points *A, B, C* is minimal.



Torricelli showed:

- If one erects equilateral triangles over the three sides of the triangle *ABC*, then their circumcircles intersect at the point sought (the so-called *Torricelli point* or *Fermat point*).

Simpson proved:

- If you connect the vertices of triangle *ABC* with the vertices of the opposite equilateral triangles, then these so-called *Simpson lines* intersect at the Torricelli point.

The multitude of activities affected SIMPSON's health and ultimately led to his early death.

After reading GUILLAUME DE L'HÔSPITAL's *Analyse des infiniment petits* [Analysis of the infinitely small] about LEIBNIZ's differential calculus, he, the author of one of the most distinguished books on NEWTON's doctrine, became aware that English mathematicians were losing touch with developments on the continent – he wrote admonishingly:

"Foreign mathematicians have, of late, been able to push their researches farther, in many particulars, than Sir ISAAC NEWTON and his followers here, have done ..."

A supplement to this biography of THOMAS SIMPSON

- *To derive SIMPSON's rule* (the $\frac{1}{3}$ *rule*)

We are looking for a quadratic function *p* with $p(x) = rx^2 + sx + t$ for which

$f(a) = p(a) = ra^2 + sa + t$, $f(b) = p(b) = rb^2 + sb + t$ and

$f(\frac{a+b}{2}) = p(\frac{a+b}{2}) = r \cdot (\frac{a+b}{2})^2 + s \cdot \frac{a+b}{2} + t = \frac{1}{4} r \cdot (a^2 + 2ab + b^2) + \frac{1}{2} s \cdot (a+b) + t$.

The last term can be transformed:

$f(\frac{a+b}{2}) = \frac{1}{4} \cdot (ra^2 + 2rab + rb^2 + 2sa + 2sb + 4t) = \frac{1}{4} \cdot (f(a) + f(b) + 2rab + sa + sb + 2t)$,

so $4 \cdot f(\frac{a+b}{2}) - f(a) - f(b) = 2rab + sa + sb + 2t$.

For the definite integral of $p(x)$ over the interval $[a, b]$ we therefore have

$$\int_a^b p(x)dx = \frac{1}{3} r \cdot (b^3 - a^3) + \frac{1}{2} s \cdot (b^2 - a^2) + t \cdot (b-a)$$

$$= (b-a) \cdot \left[\frac{1}{3} r \cdot (b^2 + ab + a^2) + \frac{1}{2} s \cdot (b+a) + t \right]$$

$$= \frac{1}{6} \cdot (b-a) \cdot \left[2rb^2 + 2rab + 2ra^2 + 3sb + 3sa + 6t \right]$$

$$= \frac{1}{6} \cdot (b-a) \cdot \left[(2ra^2 + 2sa + 2t) + (2rb^2 + 2sb + 2t) + (2rab + sa + sb + 2t) \right]$$

$$= \frac{1}{6} \cdot (b-a) \cdot \left[2 \cdot f(a) + 2 \cdot f(b) + (2rab + sa + sb + 2t) \right]$$

$$= \frac{1}{6} \cdot (b-a) \cdot \left[2 \cdot f(a) + 2 \cdot f(b) + 4 \cdot f(\tfrac{a+b}{2}) - f(a) - f(b) \right]$$

$$= \frac{1}{6} \cdot (b-a) \cdot \left[f(a) + f(b) + 4 \cdot f(\tfrac{a+b}{2}) \right]$$

- *On the development of NEWTON's approximation method*

In his work *Methodus fluxionum et serierum infinitarum* (On the method of fluxions and infinite sequences), written between 1664 and 1671, NEWTON explained the following approach to the approximate solution of a third-degree equation using the example of $x^3 - 2x - 5 = 0$.

If you insert the value 2 for *x*, you get a negative result: : $2^3 - 2 \cdot 2 - 5 = -1$.

With $x = 2 + h$: $(2+h)^3 - 2 \cdot (2+h) - 5 = 8 + 12h + 6h^2 + h^3 - 4 - 2h - 5 = h^3 + 6h^2 + 10h - 1$.

Neglecting the cubic and the quadratic term, we obtain the linear equation $10h - 1 = 0$, so $h = \frac{1}{10}$.
A first approximate solution is therefore $x \approx 2.1$.

JOSEPH RAPHSON (1678-1715) studied the general cubic equation $x^3 - bx + c = 0$, for which g is a first approximate solution. Then applied to $x = g + h$:

$$(g+h)^3 - b\cdot(g+h) + c = g^3 + 3g^2h + 3gh^2 + h^3 - bg - bh + c$$
$$= (g^3 - bg + c) + h^3 + 3gh^2 + h\cdot(3g^2 - b)$$

Neglecting the cubic and the quadratic term, we obtain the linear equation

$(g^3 - bg + c) + h\cdot(3g^2 - b) \approx 0$, so $h \approx -\dfrac{g^3 - bg + c}{3g^2 - b}$.

With $f(x) = x^3 - bx + c$ we therefore get $f(x) \approx 0$ for $x \approx g - \dfrac{f(g)}{f'(g)}$.

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https://www.spektrum.de/wissen/thomas-simpson-weber-und-wahrsager/2025139

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