SEKI KOWA (1642 – October 24, 1708)

by HEINZ KLAUS STRICK, Germany

The time of the birth of SEKI KOWA (also called SEKI TAKAKAZU) coincided with a dramatic period in Japanese history: In the mid-16th century, the country was still in a power struggle between rival princes who were violently trying to gain supremacy over the country (*shogunate*).

Firearms imported to Japan by European traders ultimately played a decisive role in the war. From 1543 onwards, merchants from Portugal, and later from other countries, had begun to trade. The merchants were followed by missionaries who converted hundreds of thousands to Christianity. Among the missionaries were Jesuits who informed Japanese scientists about the state of mathematical development in Europe.

The European merchants and missionaries tried to exert political influence. Then, in 1598, TOKUGAWA IEYASU, one of the military leaders, succeeded in defeating all his rivals and thus unified Japan by force. To reduce any possible influence of the emperor (*Tenno*), he moved the seat of government from Kyoto to his previous headquarters, a small fishing village called Edo, which was later named Tokyo (literally: eastern capital).

He expelled the foreign merchants and missionaries from the country, banned the Christian faith and had the Christian churches destroyed. Only a few Dutch merchants, who rejected any thought of missionary work, were allowed to continue their trade. An artificial, walled island was created for them at the port of Nagasaki and a merchant ship was only allowed to dock there once a year.

For over 200 years, this was Japan's only connection to the outside world, as the rulers also forbade Japanese ships from sailing to foreign countries. It was not until 1854 that American warships forcibly broke the unilateral isolation.

During the long phase of isolation, Japan experienced a cultural renaissance; painting and garden architecture developed, the famous tea ceremony came into being, as did the special way of arranging flowers (*ikebana*). Mathematics also experienced a new flowering.

Seki Kowa is considered the most important representative of *Wasan*, the Japanese mathematics of the Edo period – on his tombstone he is called an *arithmetical sage*. He was born at the end of 1642 (a few months before Isaac Newton) as the second son of a samurai warrior and as a child he was adopted by a noble family. Already at an early age, his special mathematical talent was recognised and he assisted his adoptive father in the accounting and checking of the tax levies of the district. As he had a special interest in mathematical questions, he set up his own library with Japanese and Chinese mathematics books and studied their contents intensively.

Two books by the Chinese mathematician Zhu Shijie (1260-1330) had a particular influence on Seki Kowa: *Introduction to the Study of Mathematics* and *The Precious Mirror of the Four Elements*.

In these, among other things, non-linear equations with up to four variables were set up and solved numerically (Zhu Shijie calls this the *method of the celestial elements*). To determine the solution of equations of higher degree, a method was used that has great similarity with the so-called *Horner's method* – developed 100 years later in Europe (named after William George Horner (1786-1837).

The book *Jinko-ki* (Treatise on Numbers) by Mitsuyoshi (1598-1672) from 1627 contained, among other things, approaches to integral calculus. With this book, however, a new tradition emerged: additional problems were given for which the author could not provide a solution. The formulation of unsolved problems later led to competition between Seki's school and the schools in Kyoto and Osaka.

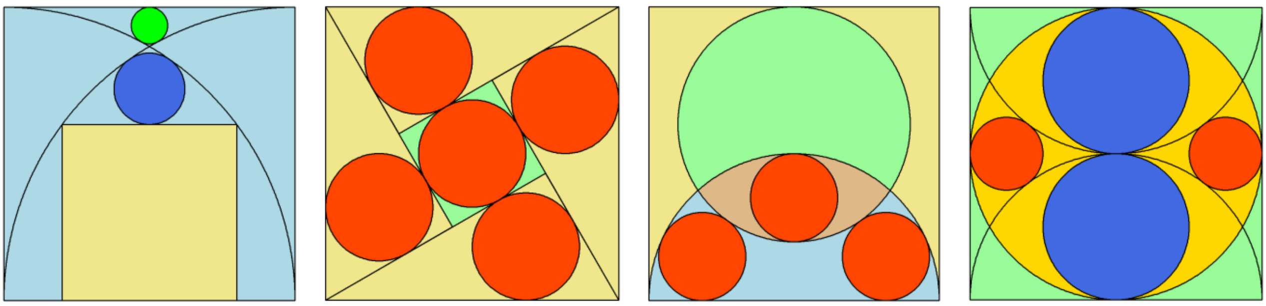
Chinese mathematics books reached Japan in the 15th century and the books were initially reprinted without changes, then annotated, and later supplemented by the respective editors'

own reflections. These books contained problems from various areas of mathematics in the form of collections of problems with solutions. In doing so, the authors explained the solution methods with a thoroughly diverse selection of topics, but without creating a "theoretical superstructure".

Books were also published that taught the use of the Japanese abacus (*Soroban*).

Also characteristic of the Wasan period were the mathematical tables of "temple geometry" (*sangaku*), on which geometric problems with inscribed or reinscribed circles, ellipses, squares, rhomboids and triangles were written down and spatial problems also appeared.

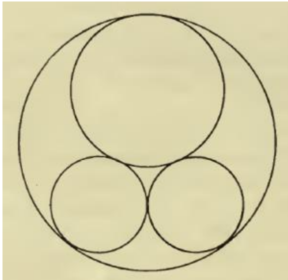
These artfully created boards were hung at Buddhist temples or Shinto shrines as offerings – as thanks to the gods for the enlightenment of having discovered and solved this problem and they serve as an intellectual challenge for visitors.



Graphics from H. K. Strick (2021): *Mathematics is beautiful*, Springer

In 1670, a book by Sawaguchi Kazuyuki was published in Osaka, which dealt, among other things, with 150 problems for which Mitsuyoshi could not give a solution. Sawaguchi Kazuyuki succeeded in solving 135 of the 150 problems and he described the remaining 15 problems as "actually unsolvable". One of the "unsolvable" problems was:

- *Three other circles are inscribed in a circle and the remaining area has 120 area units. The common diameter of the two small circles is 5 units of length smaller than the diameter of the third circle. What is the diameter of the circles in the figure?*



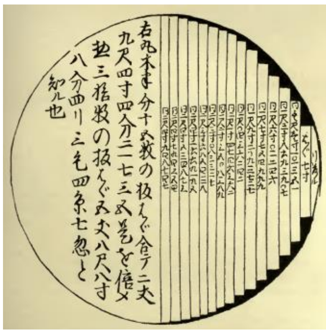
Seki Kowa achieved fame not least through the book *Hatsubi Sampo*, which he published four years later. Among other things, it contained the solutions to all the remaining 15 problems.

In the book, he presented algebraic terms with new, self-invented notations for powers. In his solutions, Seki tried to keep the actual solution methods (or even the ways to find them) hidden from competitors – as was also common in Europe. What was important was only that one could solve a problem with a new method developed by oneself – not why this method was suitable to solve the problem. Thus, the solution methods he actually adopted only become known decades after his death.

In scientific literature, Seki was often compared – in terms of his importance – with Newton. The methods developed by him (or possibly by his student Katahiro Takebe) went far beyond what was found in Zhu Shijie. His books contained generalised schemes for (numerically) solving arbitrary algebraic equations. To solve systems of linear equations with three variables, he used a method by which the solutions were obtained from tables of the equations' coefficients – comparable to the determinant method discovered ten years later by Gottfried Wilhelm Leibniz (1646-1716).

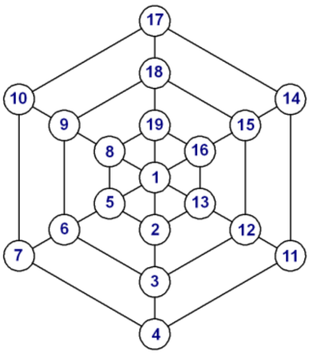
Seki also described the laws of permutation that apply to determinants. He gave formulae for the sum of the first *k* powers of the natural numbers – thus finding the *Bernoulli numbers* before Jacob Bernoulli (1655-1705).

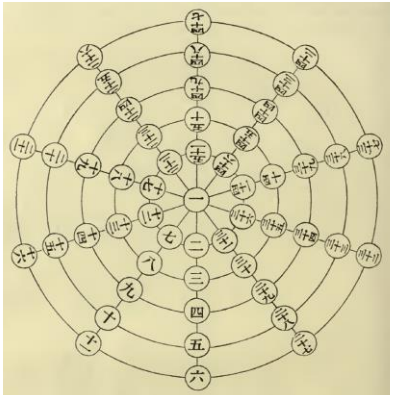
Seki and Takebe calculated the circle number π according to the so-called *Enri* method (*en* means a circle and *ri* means a theory), a peculiar, unusual procedure: Here, infinitesimal pieces of arcs over chords were considered, which were calculated step by step with increasing accuracy – the determination of ten digits was done by series expansion.



Finally, in Seki's books, one also finds a wealth of tasks on recreational mathematics, for example, procedures on how magic squares or magic circles can be generated.

12	11	10	45	46	49	2
47	20	19	35	37	14	3
44	34	24	29	22	16	6
7	17	23	25	27	33	43
8	18	28	21	26	32	42
9	36	31	15	13	30	41
48	39	40	5	4	1	38





Graphic from David Eugene Smith & Yoshio Mikami (1914):
A History of Japanese Mathematics, Chicago

It was not until 1868 that the *Wasan* books were replaced in Japan by books in the Western style – i.e. with the definition-theorem-proof scheme that had been in use since EUCLID.

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