GOTTFRIED WILHELM LEIBNIZ (July 1, 1646 – November 14, 1716)

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GOTTFRIED WILHELM LEIBNIZ was the son of a professor of law and moral philosophy at the University of Leipzig who died when Gottfried was six years old. Raised by his mother, the boy had from an early age access to his father's library, and the young Gottfried made use of it to teach himself Latin. At 15, he began studies in philosophy at the University of Leipzig.

At age 20, he sought to obtain a doctor of law degree, but the university professors declared that he was too young, and so he went to Nuremberg, where he was awarded the degree. He then entered into the service of the Archbishop-Elector of Mainz as a legal adviser.

In his travels as a diplomat to France and England, he came in contact with the most prominent scientists and mathematicians of the time. He became a friend of CHRISTIAAN HUYGENS. His first mathematical publications appeared in rapid sequence. In 1673, he was inducted into the *Royal Society*. He sought in vain to obtain a permanent position in the *French Academy of Sciences*. In the course of his applications, he did, however, introduce a calculating machine of his own invention, one that could perform even multiplication and division.

In 1676, he entered into the service of the duke of Brunswick and Lüneburg in Hanover. One of his principal duties was to research the history of the House of Guelph (Welf) in order to legitimate its claim to the English throne. When in 1714, the duke of Hanover actually became the king of England and his entire court moved to London, LEIBNIZ was left behind in Hanover to continue his researches on the House of Guelph.

LEIBNIZ took advantage of his service in the House of Hanover to make long journeys throughout Europe. Moreover, he corresponded with a great number of scientists. In the LEIBNIZ archive can be found documentation of over fifteen thousand letters that he wrote himself to over one thousand correspondents and more than twenty thousand letters addressed to him.

There is scarcely a subject that did not interest him. As he said of himself, "*When I awake, I already have so many ideas that the day's length is insufficient even to write them down.*"

Thus, for example, he busied himself trying to solve the problem of the flooding of silver mines in the Harz region and developed ideas for the construction of a submarine and for improvements in the security of door locks. He proposed a system of pensions for widows and orphans and also dispensed medical advice.

LEIBNIZ became involved in an effort to achieve a reconciliation between the Catholic and Protestant camps; he worked out plans for coinage reform in order to simplify commerce. Not least, he encouraged the creation of learned academies on the French and English model in a number of European countries: Prussia, Saxony, Russia, and the Habsburg dominions.

Above all, however, he published numerous papers on topics in mathematics, physics, and philosophy. Despite the great respect in which he was held in scientific circles, he suffered from a lack of self-esteem. Perhaps this was due to his physical appearance; perhaps he was ashamed of his strong Saxon accent.

Moreover, the dukes of Hanover valued the accomplishments of the polymath genius LEIBNIZ not at all. At the end of his life, his health suffered from the stress of the priority dispute over the infinitesimal calculus: ISAAC NEWTON had begun in 1666 to develop his *fluxion calculus* (differential calculus), but his first publications on the subject appeared only in 1687.

On his part, LEIBNIZ published his *Calculus* independently of NEWTON in the year 1684 in the article *Nova methodus pro maximis et minimis, itemque tangentibus, qua nec fractas, nec irrationales quantitates moratur, et singulare pro illi calculi genus.* This work contains all the derivative rules, including the chain rule, as well as conditions for the existence of extrema and points of inflection.

Two years later, there followed *De geometria recondita*, in which the integral sign ∫ was used. On the Continent, the notation and terminology (such as constant, variable, function) developed by LEIBNIZ spread quickly. His adherents, above all JOHANN and JAKOB BERNOULLI, proved the value of LEIBNIZ's notation in their works on calculus and its applications to physics.

In 1677, NEWTON attacked LEIBNIZ, accusing him of having "stolen" his methods. The conflict escalated over the years and created a schism in the scientific world. In 1713, a partisan commission appointed by the *Royal Society* upheld the charges of plagiarism.

Today, however, it is generally acknowledged that the two theories were developed independently of each other.

LEIBNIZ's ability to choose a suitable symbolism for dealing with scientific questions is seen as well in his extensive studies in formal logic. He discovered the significance of the binary number system, on which the modern computer is based. But he linked it as well to his theological-philosophical worldview: In the credo *Without God there is nothing*, he gives the value 1 to God and 0 to nothing. His famous and widely ridiculed thesis that the world is the *best of all possible worlds* is not a sign of religious naivety, but expresses his conviction that this world contains so much potential for development that its current state can always be improved.

In mathematics texts one finds the name Leibniz in numerous connections:

- The LEIBNIZ product rule tells how to form the higher derivatives of a product of functions:

$$(f\cdot g)^{(n)} = \binom{n}{0}\cdot f^{(n)}\cdot g^{(0)} + \binom{n}{1}\cdot f^{(n-1)}\cdot g^{(1)} + \binom{n}{2}\cdot f^{(n-2)}\cdot g^{(2)} + \ldots + \binom{n}{n}\cdot f^{(0)}\cdot g^{(n)}$$

- The LEIBNIZ criterion is an assertion about series with alternating positive and negative terms (alternating series): If the terms of the series $(a_n)_{n\in IN}$ have limit zero, then the series $\sum_{n=0}^{\infty}(-1)^n a_n$ converges.
- As early as 1674, LEIBNIZ used the power series development of the arctangent function to derive what is today called the LEIBNIZ series for the number π: $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-+\ldots=\frac{\pi}{4}$.

LEIBNIZ did not provide proofs for many of the rules that he discovered. Many of his justifications seem today rather thoughtless and careless. However, they show a keen ability to discover valid relationships. For example, from the series development $\frac{1}{1+x}=1-x+x^2-x^3+x^4-+\ldots$

one obtains by integration $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-+\ldots$

This equation is valid for $|x| < 1$. If one simply inserts the value 1 for the variable *x*, one then obtains the correct limiting value ln(2) ≈ 0.693, which is something quite remarkable.

Also LEIBNIZ's method of determination of the limit of the series of reciprocal triangular numbers, with which LEIBNIZ became interested in 1673, contains an invalid line of reasoning:

He argues that $1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\ldots=2$

because

$$(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots)-1+\frac{1}{2}\cdot(1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\ldots)$$
$$=(\frac{1}{2}+\frac{1}{2})+(\frac{1}{3}+\frac{1}{6})+(\frac{1}{4}+\frac{1}{12})+(\frac{1}{5}+\frac{1}{20})+\ldots=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$$

and therefore $-1+\frac{1}{2}\cdot(1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\ldots)=0$.

LEIBNIZ calculates with infinite as with finite numbers. The sum is infinite, and so the difference of infinite quantities is indeterminate. In this case, the result happens to be correct.

It was only a century later that mathematicians such as AUGUSTIN CAUCHY put the foundations of analysis on a firm footing.

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