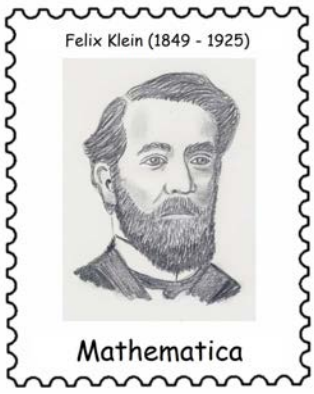
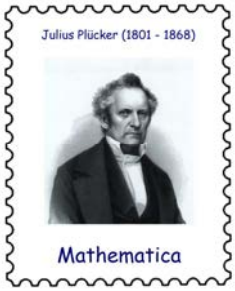
FELIX KLEIN (April 25, 1849 – June 22, 1925)

by HEINZ KLAUS STRICK, Germany



(drawing: © Andreas Strick)

Even the date of his birth has a mathematical slant to it – FELIX KLEIN, the son of a Prussian civil servant, born in Düsseldorf, liked to point out that the date of his birth consisted of three perfect squares (2^2, 5^2, 43^2). Every publication on the life and work of KLEIN emphasizes his vital importance for the development of mathematics both as a rigorous discipline and as a pedagogical subject, and points in particular to his success in the organization of academic life. Nevertheless, he does not (yet) belong among the company of important figures who have been commemorated with a postage stamp.

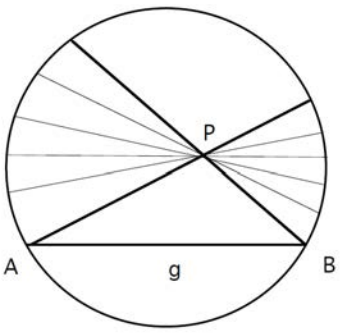


After completing secondary school, the sixteen-year-old FELIX KLEIN began his university studies in mathematics and experimental physics in Bonn, completing his doctorate at the age of 19 with a dissertation on JULIUS PLÜCKER's "line geometry". He continued to do research and publish on questions in geometry, such as the further development of the notion of coordinates and the dimension of a space.

In 1870, he had to break off a study trip to Paris, where he was under the tutelage of CAMILLE JORDAN, following the outbreak of the Franco-Prussian War. In 1871, he completed his habilitation under ALFRED CLEBSCH in Göttingen, where he then became a senior lecturer (Privatdozent) before accepting a professorship in Erlangen in 1872, at the age of 23.

His paper *Vergleichende Betrachtungen über neuere geometrische Forschungen* (A comparative review of recent researches in geometry), a sort of research programme cum manifesto, entered the history of mathematics under the rubric of KLEIN's *Erlangen Programme*. Since the discovery of non-Euclidean geometries at the beginning of the nineteenth century by NIKOLAI IVANOVICH LOBACHEWSKY und JÁNOS BOLYAI, researchers had been pursuing the question whether there is only one geometry or several. KLEIN showed, with the help of group theory, that all of these geometries could be subsumed under projective geometry as special cases.

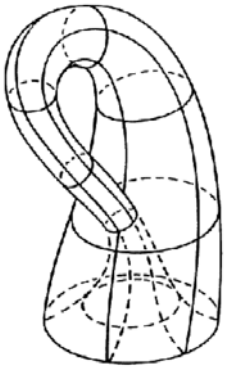
In his paper, KLEIN used the term *hyperbolic geometry* for the non-Euclidean approaches of BOLYAI und LOBACHEWSKY, *elliptic geometry* for *spherical geometry*, and *parabolic geometry* for Euclidean geometry – terms that have since become standard. Inspired by a publication by the Italian mathematician EUGENIO BELTRAMI, KLEIN developed a model of *hyperbolic geometry* in which for every straight line and point external to the line, there are at least two lines that pass through the given point and are parallel to the given line.



The plane can then be modelled as the interior of a circle, and chords AB inside the circle (excluding the endpoints lying on the circle) are the "lines" of this geometry. Then for every line AB and for every point P external to this line, there are arbitrarily many parallel lines, namely all lines that do not pass through the triangle ABP.

In 1875, FELIX KLEIN moved to the *Technical University*, Munich (Technische Hochschule München). There, he instituted a required four-semester series of courses called higher mathematics for students of engineering, an innovation that was soon adopted by the other technical universities in Germany.

In 1880, KLEIN was called to a chair in geometry at the University of Leipzig. Within a short period of time, he implemented the idea of establishing a mathematical seminar, that is, a separate building in which lectures are held containing its own library, including a collection of models depicting aspects of geometry. Soon, similar facilities were established at other universities. Here as well, he instituted required courses for beginning students. These were enhanced with tutorials. He had already promoted the idea of a collaboration between university and industry in his inaugural lecture in Leipzig, *Über die Beziehungen der neueren Mathematik zu den Anwendungen* (On the relationships between today's mathematics and its applications), in which he also proposed a chair in *applied mathematics*.



In 1882, he discovered a geometric object between whose interior and exterior it was impossible to distinguish; today, it is known as the *KLEIN bottle* (perhaps on account of a mistaken translation into English, resulting from a confusion of a single letter, between the words Flasche (bottle) and Fläche or Flaeche (surface).

(former Wikipedia picture)

KLEIN was now at the peak of his creative powers. In his paper *On RIEMANN's Theory of Algebraic Functions and their Integrals*, he pointed out connections between various branches of mathematics. In his investigation of the mappings of the regular icosahedron to itself, he discovered connections with the problem of the algebraic solution of equations of the fifth degree. (In a lecture on this subject, he mentioned the smallest noncyclic group, which today in his honour is known as the *KLEIN four group*: it is the group of four elements that can be interpreted, for example, as the group of symmetries of a rectangle.)

	1	a	b	ab
1	1	a	b	ab
a	a	1	ab	b
b	b	ab	1	a
ab	ab	b	a	1

In fall 1882, however, he suffered a physical and emotional breakdown: what triggered it appears to have been a professional and personal quarrel with the French mathematician HENRI POINCARÉ.

POINCARÉ had named a certain class of mappings *Fuchsian functions*, although unbeknownst to him, it was KLEIN and not LAZARUS FUCHS who had discovered them. The correspondence between these two outstanding mathematicians, which had begun on a strictly professional level, quickly developed into a bitter conflict between two rivals. When POINCARÉ then proceeded to publish new and far-reaching results in non-Euclidean geometry, which was the area of KLEIN's specialization, this touched off for KLEIN a creative crisis. In one phase of his depression, he decided to end all mathematical research activity in order to free himself of the pressure of competition.

FRIEDRICH ALTHOFF, an influential official in the Prussian Ministry of Culture, was able to motivate KLEIN to resume his research by making him an attractive offer: In 1886, KLEIN moved to the University of Göttingen. Thanks to the fruitful collaboration between KLEIN und ALTHOFF regarding this new position, the university soon became the most important centre for mathematics in the world. Together with HEINRICH WEBER, KLEIN founded a *Mathematical Association* (Mathematische Gesellschaft), which assembled weekly to hear a lecture. He encouraged all the lecturers to write up their work and make it available in the department's reading room.

KLEIN was also one of the founders of the *German Mathematical Society* (Deutsche Mathematiker-Vereinigung, 1890). He took over the editorship of the journal *Mathematische Annalen* and developed it into one of the most prestigious mathematical journals in the world.

Beginning in 1893, women were allowed to study mathematics at Göttingen. Among the 48 doctoral students who received their PhD's under the direction of KLEIN was GRACE CHISHOLM YOUNG, the first woman to receive a doctorate in mathematics – indeed in any field – in Germany.

KLEIN had a particular interest in the education of teachers of mathematics and the manner in which mathematics was taught in the schools. He wrote a three-volume work, *Elementary Mathematics from an Advanced Standpoint*, whose goal was to help teachers to bridge the gap between the subject matter taught at university and the usual topics that were covered in the schools and to integrate them into the curriculum. He was vehement in his belief in the need to strengthen those aspects of instruction that gave students the capacity to think in three dimensions. He promoted education in functional thinking and influenced the treatment of differential and integral calculus as a unified topic of instruction in secondary school (this point of the Merano curriculum blueprint of 1905, largely influenced by KLEIN, was, however, not implemented until 1925). KLEIN and ALTHOFF saw to it that the first chair in mathematical pedagogy was established in Göttingen. And not least, it is to KLEIN that can be attributed that in the reform of the German educational system of 1900, education in mathematics and the natural sciences was placed on a par with the humanities. Indeed, the ideas that he promoted as head of the *International Commission on Mathematical Education* took KLEIN's influence beyond the borders of Germany.

Following his retirement in 1913, FELIX KLEIN, who in an appreciation was dubbed the "pope and foreign minister of mathematics," did not fade into obscurity. Thanks to his wide-ranging contacts, he was able to engage prominent mathematicians to contribute to the *Encyclopädie der mathematischen Wissenschaften mit Einschluß ihrer Anwendungen* (Encyclopaedia of mathematics including applications). He also wrote out his lectures, including *Über die Entwicklung der Mathematik im 19. Jahrhundert* (On the development of mathematics in the nineteenth century).

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https://www.spektrum.de/wissen/felix-klein-1849-1925/1157014

Translated by David Kramer

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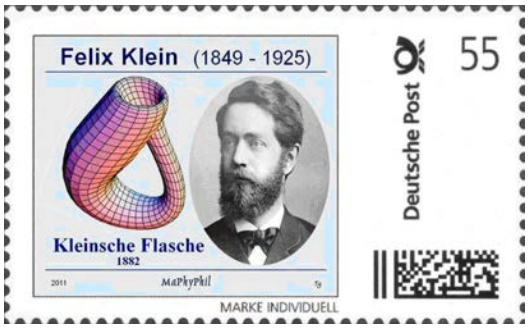
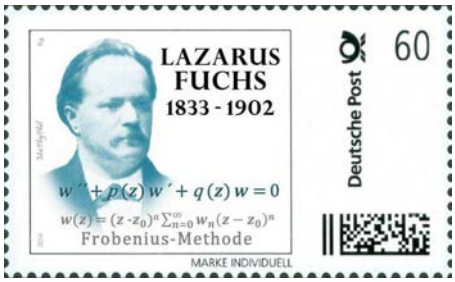
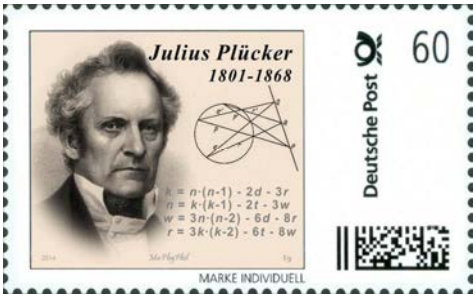
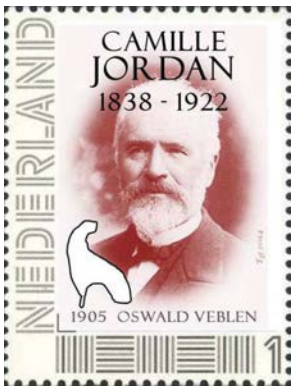
Here an important hint for philatelists who also like individual (not officially issued) stamps:









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