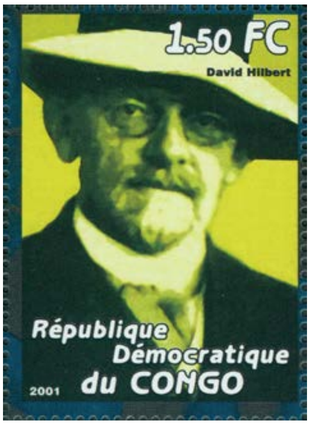
DAVID HILBERT (January 23, 1862 – February 14, 1943)

by HEINZ KLAUS STRICK, Germany

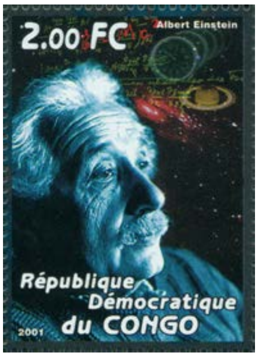


It is a mystery why the German postal authorities have never been moved to commemorate DAVID HILBERT, one of the most important mathematicians of all time.

However, for the *Democratic Republic of the Congo* (formerly *Zaire*), HILBERT was named one of their twelve *important people of the twentieth century*.

What is unusual about HILBERT is the fact that over a long scientific career, he worked in a number of very different areas of mathematics, and each creative period resulted in outstanding publications.

After opening up the field of invariant theory with new approaches and insights, he went on in the 1890s to make important contributions to algebraic number theory, and then from 1899 onward was deeply involved with the *Foundations of Geometry*, which became the title of one of his publications.



In the second half of the first decade of the twentieth century, he did research on questions in mathematical analysis; in his honour, certain vector spaces are known today as HILBERT spaces. These structures, defined purely mathematically, astonishingly found later application in quantum mechanics.

Beginning in 1910, he focused on problems in mathematical physics; his article on the general theory of relativity appeared a few days after—though it in fact had been submitted earlier—the paper on the same topic by ALBERT EINSTEIN, without his having any idea of getting into a dispute with Einstein as to who had been first to come up with the idea.

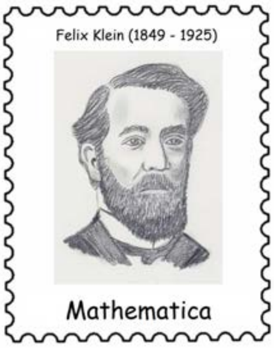
In his last decades (from 1918 on), he worked on the foundations of mathematics, dealing in particular with questions about formalization (HILBERT's *programme*); his research contributions were published in two volumes, in 1934 and 1939.

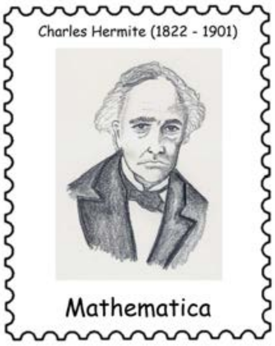
DAVID HILBERT was born and raised in Königsberg (East Prussia, today Kaliningrad, Russia); a year before his final high-school examinations in 1880, he changed schools, going from the more classically oriented *Collegium Fridericianum*, the school from which IMMANUEL KANT graduated, to the *Königliche Wilhelms-Gymnasium*, which had a stronger orientation toward mathematics and the sciences. Even though HILBERT achieved the highest marks in mathematics, that subject did not play a special role for him during his school years, for as he attested in a memoir, it was "*… because I knew that I would be doing it later*."

His father, a solicitor, was unconvinced that the career of a mathematician offered much promise professionally, but he let his son do as he liked.

After five years of study at the University of Königsberg, known familiarly as the *Albertina*, he received his doctorate under the supervision of FERDINAND LINDEMANN (who had become world famous for being the first to prove the *transcendence* of the number π, a result that had as a corollary a proof that the classical problem of "squaring the circle" with straightedge and compass alone was impossible).

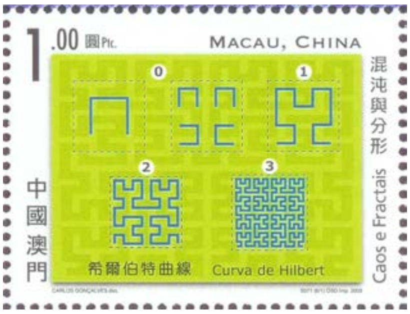
After short study journeys to Leipzig (FELIX KLEIN) and Paris (HENRI POINCARÉ, CHARLES HERMITE), he completed his habilitation in Königsberg, where he became a privatdozent (lecturer), and at age 30, an assistant (non-tenured) professor.





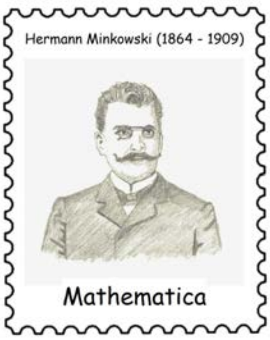
(drawings: © Andreas Strick)

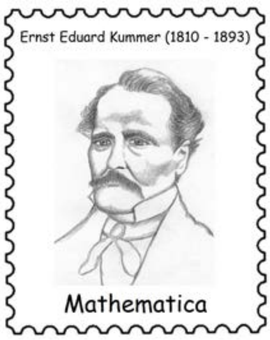
In 1891 he discovered a curve that today is called the Hilbert curve: It is defined by recursion and it has the surprising property that it is continuous and that its range completely fills a square.

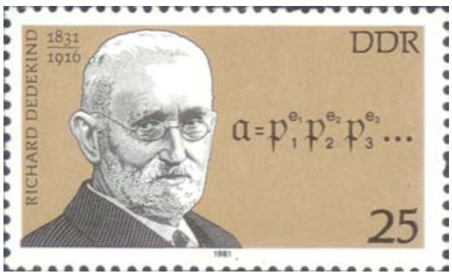


In 1895, he was offered a chair at the University of Göttingen, which from the time of Carl Friedrich Gauss had become one of the most important centres of mathematical research in Germany. Hilbert made Göttingen *the* international centre for mathematics and physics. Despite numerous offers from other universities, Hilbert remained in Göttingen until the end of his life. From 1933, he was forced to experience the destruction of his life's work by the racial politics of National Socialism.

In 1897, the German Mathematical Society asked Hilbert to provide a summary of the current state of research in algebraic number theory. As a result of this proposal, he wrote, in collaboration with his university friend Hermann Minkowski (at the time still in Königsberg, later in Göttingen), the famous *Zahlbericht* (number report), in which he systematized the theories of Ernst Eduard Kummer, Leopold Kronecker, and Richard Dedekind, and not only with regard to notation, for he also found and mended holes in some of their proofs.







After completing the *Zahlbericht*, Hilbert set about placing Euclidean geometry on a solid axiomatic foundation. He formulated a complete system of twenty axioms from which all the theorems of geometry in three-dimensional space could be derived (in the strictest logical sense). While Hilbert used such descriptive notions as point, line, and plane, he asserted that they could just as well be replaced with terms such as *table, chair,* and *beer mug*.

These geometrical objects were described by

– eight *axioms of combination, or incidence*
  (for example, (I.1): *Two distinct points P and Q always determine a line g*),

– four *axioms of order*
  (for example, (II.1): *If B lies between A and C, then B also lies between C and A*),

– five *axioms of congruence* (III),

– the *parallel axiom* (IV)
  (Let *g be an arbitrary line, and P a point external to g; then in the plane determined by g and P there is at most one line g´ that passes through P and does not intersect g*),

as well as two *axioms of continuity* (for example, the Archimedean axiom (V.1)

> (If *AB and CD are line segments, then there exists a number n such that if one lays the segment CD on the ray beginning at A and proceeding in the direction of B, then lays the segment CD again on this ray, placing the point C where the point D was, and proceeding in this manner n times, the nth such segment will contain the point B*.)

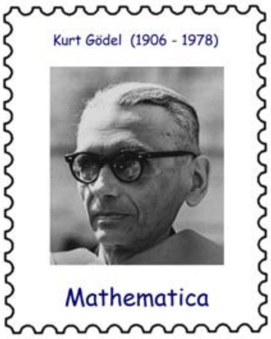
By 1900, HILBERT's reputation had reached a high point. He was elected president of the *German Mathematical Society* and was invited to give one of the plenary lectures at the *Second International Congress of Mathematicians*, held in Paris that year. In that lecture, he offered a programmatic overview of the important research topics in mathematics for the new century:

- *What will be the particular goals toward which the leading mathematical minds of the coming generation will strive?*
- *What new methods and new facts will the new centuries discover – in the broad and rich fields of mathematical thought?*

His list of *23 unsolved problems* influenced research in the twentieth century to a large degree, and anyone solving one of HILBERT's was assured of immediate renown. HILBERT was convinced that every mathematical problem could ultimately be solved:

> [*Solvability*] *is for us a powerful spur while we are working on a problem; we hear within us a constant encouragement: Here is the problem; seek the solution. You can find it through pure reasoning, for in mathematics, there is no Ignorabimus!*

This optimism was greatly damped over the course of the coming years: In the second of his 23 problems, HILBERT had addressed the issue of a *consistent axiomatization of arithmetic*. In 1889, GIUSEPPE PEANO had published an axiomatic system for arithmetic, but it had not been shown to be free of contradiction, that is, it had not been proven *consistent*. (By *consistency* is meant the impossibility of being able to derive within a given axiomatic system some statement as well as its negation.)



In a lecture in 1928 at a mathematical convention in Bologna, HILBERT presented a logical calculus of formal inference. He hoped that this would form the basis of a proof of consistency.

But then in 1931, the Austrian logician KURT GÖDEL proved that a proof of consistency of a theory cannot be accomplished solely with the methods of that theory.

(Portugal: Mathematicians of the century: Henri Poincaré, Kurt Gödel, Andrei Kolmogorov)

Hilbert's first problem also turned out to be *undecidable*: In 1878, Georg Cantor had proposed the *continuum hypothesis*: *There exists no set whose cardinality lies between the cardinality of the natural numbers and the cardinality of the real numbers*. In 1925, John von Neumann, one of Hilbert's 69 doctoral students, produced a consistent axiomatic construction of set theory (Hilbert: *May no one succeed in driving us from the paradise that* Cantor *has created for us*).

(Portugal: Computer scientists of the century: Alan Turing, John von Neumann)

But in the 1960s, Paul Cohen showed that the continuum hypothesis cannot be derived from the axioms of set theory, so that it may be taken as a supplementary axiom, though its negation could equally well be taken.

Hilbert's death a few days after the German defeat at Stalingrad received little public notice. But there were a number of memorial services in the USA, where most of his students were now working. Hilbert's gravestone in the Göttingen municipal cemetery bears his optimistic words: *We must know. We shall know*.

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https://www.spektrum.de/fm/976/749955.4361920.pdf

Translated by David Kramer

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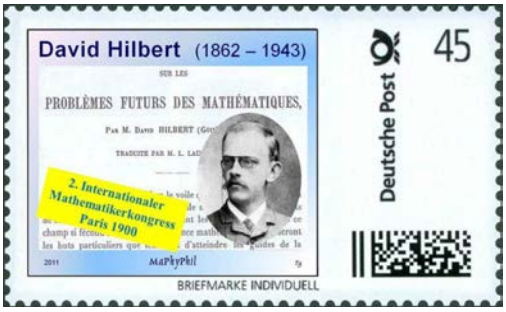
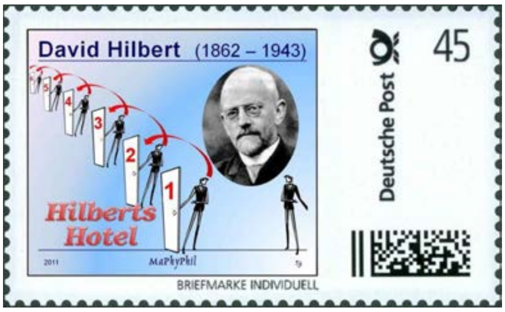
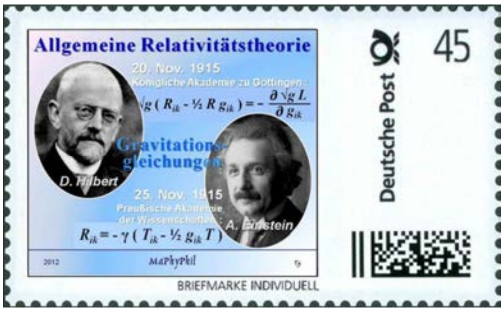
Here an important hint for philatelists who also like individual (not officially issued) stamps:







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