ERATOSTHENES OF CYRENE (276 BC - 194 BC)

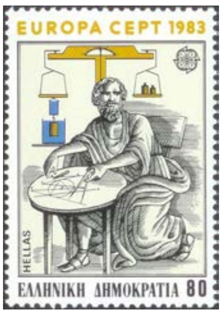
by HEINZ KLAUS STRICK, Germany

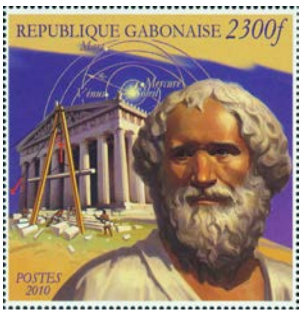
Born in the Greek settlement of Cyrene on the African Mediterranean coast (today Libya), ERATOSTHENES received a comprehensive education, particularly in Athens. He made a name for himself there as a scholar at an early age. He spent most of his life – until his death – in Alexandria, Egypt.

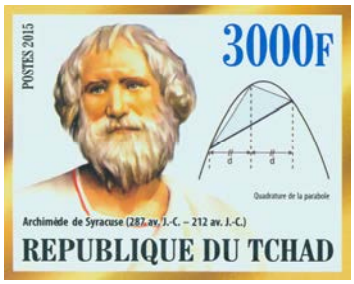
The pharaoh PTOLEMAIOS II had taken over joint rule of the kingdom in 285 B.C., and then, after the death of his father PTOLEMAIOS I (one of the generals of ALEXANDER THE GREAT and founder of the Diadochic Empire) he became the sole ruler in Egypt, making the port city of Alexandria an important center of the ancient world. The library located in the palace district *Museion* with (estimated over) 500,000 scrolls developed over the years into the largest of the Hellenistic cultural area. Among the leaders of this library was the poet CALLIMACHOS, one of the teachers of ERATOSTHENES.

After his father's death in 246 B.C., PTOLEMY III succeeded him on the throne of the pharaohs. He entrusted the education of his son PHILOPATOR to ERATOSTHENES; and after CALLIMACHOS's death he also appointed him head of the library.

In the assessment of his importance as a scientist, one occasionally finds the term *pentathlon* about ERATOSTHENES, as a pentathlete who, although he performs better than average in various fields, each of them is not the most outstanding in their respective area. Even the somewhat mockingly used term *Beta*, i.e. *second*, only seems to classify him as a second-rate scientist, but it only means that he was second behind the *Alpha* of his time, and that meant his friend ARCHIMEDES.

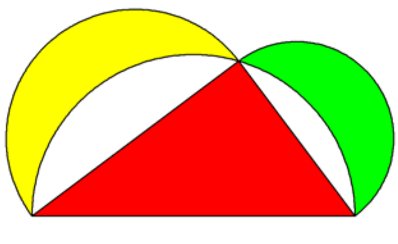




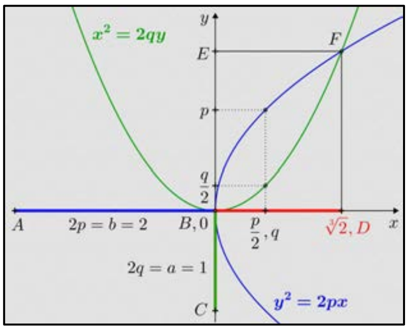


ERATOSTHENES is regarded as one of the mathematicians of antiquity who succeeded in solving the so-called *Delian problem*: According to legend, the oracle of Delos had advised the inhabitants of the island to double the volume of the cubic altar in the Temple of Apollo in order to prevent a plague epidemic. The mathematicians of antiquity already suspected that it was not possible to construct the altar's side length with compass and ruler alone. The French mathematician PIERRE-LAURENT WANTZEL would prove this in 1837.

HIPPOCRATES OF CHIOS (whose "Lune" – the crescent shaped area between two circles – amazed the mathematical world at that time and still fascinates us today, see on the right) found that a solution of the equation $b^3 = 2a^3$ is possible if it is possible to construct intermediate quantities *x* and *y* (so-called *mean proportionals*) which satisfy the equation $a : x = x : y = y : b$ with $b = 2a$. Then $x = a \cdot \sqrt[3]{2}$ for the quantity *x*.



MENAECHMUS (around 380 BC – 320 BC) solved the problem with the help of two parabolas with $a = 2q = 1$ and $b = 2p = 2$, see on the right.



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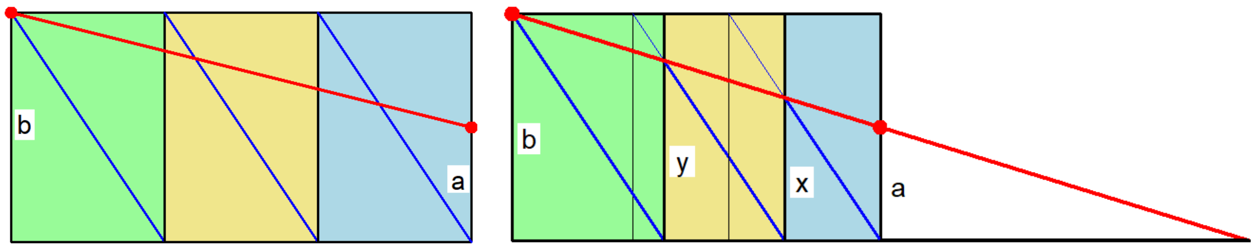
ERATOSTHENES developed a mechanical solution, of which he was so proud that he had a plaque attached to a column in the temple. However, his ingenious idea earned him malicious comments from some contemporaries, as the mechanical solution was considered an unworthy method for a mathematician.

The apparatus he called the *Mesolabium* consists of three thin rectangular plates of equal size, each of which has a diagonal marked on it and can be moved horizontally against each other, so that they can also lie partially on top of each other. The upper left corner of the left-hand panel is connected to the centre of the right-hand side of the right-hand panel (for example, by placing a ruler on it). Then the two right panels are moved to the left so that their diagonals intersect with the "movable" line at the left edges, see below. One then reads off distances in the resulting figure which fulfil the above-mentioned equation of proportion.



Inseparably connected with the name of ERATOSTHENES is an algorithm, which probably does not originate from him, but his designation as *sieve* does. In a table with natural numbers up to a maximum value *n*, all numbers greater than 2 that are divisible by $p_1 = 2$ are successively deleted.

The smallest natural number not deleted is then the prime number $p_2 = 3$, its multiples are deleted in the second step. The procedure is continued with the next found prime numbers $p_3 = 5$, $p_4 = 7$, … up to a prime number $p_k < \sqrt{n}$. All numbers that have not yet been deleted have the property of being prime numbers.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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61	62	63	64	65	66	67	68	69	70
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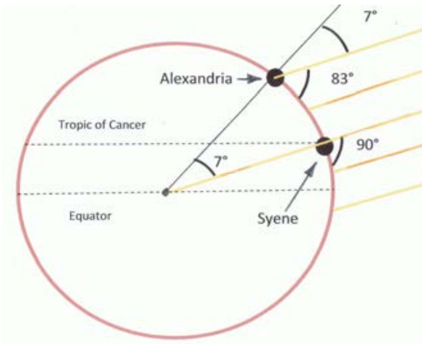
The achievements of ERATOSTHENES in astronomy are also worthy of special mention. With great care he compiled a star catalogue including the brightness of the stars and supplemented this with a collection of stories about 44 constellations. He determined with astonishing accuracy the *obliquity of the ecliptic*, i.e. the angle of inclination between the plane in which the Sun appears to move in the course of a year and the equatorial plane of the Earth. To ensure that the feast of the goddess Isis always fell on the same day of the year, he suggested to his ruler – 193 years before the Julian calendar reform – that a leap day should be introduced every four years.

ERATOSTHENES became famous for his method of determining the circumference of the globe. Based on the data collected in the library he ran, he knew that the distance between the cities of Alexandria and Syene (on the Tropic of Capricorn, now Aswan) was about 5000 "stadia". Assuming that both cities are on the same longitude and that the sun's rays arrive at the two places parallel to each other (which is not exactly true for either), he had the angle of the sun's elevation measured at noon on the day of the summer solstice:

In Syene the sun was at its zenith, in Alexandria it was 7° 12' away from it, which is a fiftieth of 360°. Hence the distance between the two cities is therefore also one fiftieth of the Earth's circumference. Unfortunately, we do not know what modern measure corresponds to the length of a "stadium".

But if we look at the actual distance between the two places (835 km), the circumference of the earth is approximately 41,750 km – a deviation of only 4.2 percent from the true value.



(source: julian.trubin.com)

ERATOSTHENES can be considered the "inventor" of geography ("geo" = earth, "graphein" = to draw). In his three-volume work, in which he outlined his findings on the spherical shape of the earth and the determination of the earth's circumference, he set the standards for a new science. As well as the description of landscapes, he wanted to collect as much concrete data as possible. He himself compiled a list with the names of 400 cities of the *Oikumene* (= the civilized or inhabited world) with information about their location with the help of a cartographic network as well as their affiliation to existing states. The knowledge written down in his work he took – quite critically – from the numerous travel reports collected in the library.

The polymath also wrote poetry and wrote writings on philosophy, music theory and history. He was the first to attempt to compile a chronology of documented historical events and to summarize them into epochs.

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https://www.spektrum.de/wissen/der-erfinder-der-geographie/1646030

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