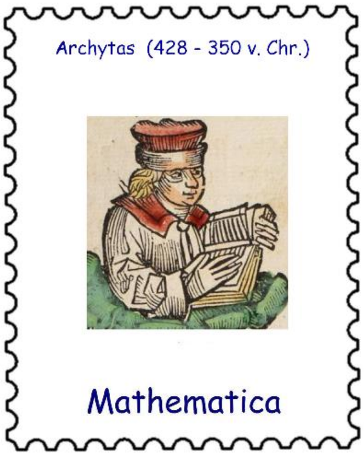
ARCHYTAS OF TARENTUM (428 BC – 350 BC)

by HEINZ KLAUS STRICK, Germany



In the 5th century BC, the city of Tarantum developed into the most important city in Magna Graecia, the region of southern Italy populated by Greeks. There, the followers of the religious-philosophical school of the Pythagoreans had found refuge – they had gradually been driven out of all other cities.

(Picture from the Nuremberg Chronicle of 1493)

In the Peloponnesian War (431-404 BC), Tarantum and Syracuse (Sicily) were allies of Sparta in the struggle against Athens. When the tyrant DIONYSIOS II took power in Syracuse and attacked other cities in southern Italy, the rulers in Tarantum managed to keep their city out of the warring conflicts.

Probably the most important politician and commander-in-chief of the Tarentine forces was the mathematician and philosopher ARCHYTAS. This clever statesman, who was keen to strike a balance, was confirmed in office seven times in succession, although a law in force at the time did not actually permit immediate re-election.

Among ARCHYTAS's friends was the philosopher PLATO, founder of the *Academy*, the first school of philosophy in Greece.

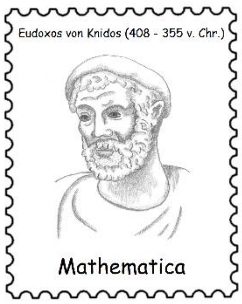
In 361, PLATO was suspected of high treason during a stay in Syracuse, but was able to leave the city unharmed thanks to ARCHYTAS's good relations with DIONYSIOS II.

For the convinced Pythagorean ARCHYTAS, mathematics was the basis of all sciences. The division of the basic mathematical sciences of the *quadrivium* (literally: four ways), which was common in the Middle Ages, goes back to him: arithmetic (number theory), geometry, astronomy and music.

ARCHYTAS became famous, among other things, because he found a solution to the so-called *Delian problem*.

According to legend, the citizens of the island of Delos had turned to the oracle at Delphi to find out how they could defeat a plague sent by APOLLO. The oracle answered them that they would have to double the size of their cube-shaped APOLLO altar in order to be spared from further deaths.

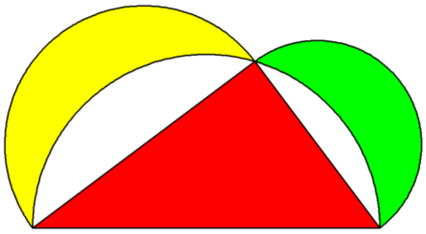
Since this answer seemed strange to the inhabitants of Delos, they turned to PLATO, who advised them to look into mathematics to solve the problem of doubling the volume of a given cube.

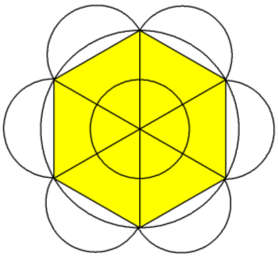


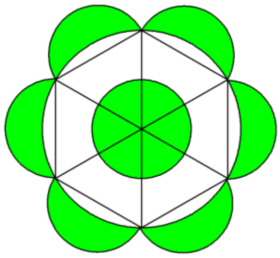
Specifically, PLATO advised them to consult ARCHYTAS OF TARENTUM, EUDOXOS OF KNIDOS and MENAECHMUS. All three mathematicians found a different solution method, which PLATO rejected, however, because as mechanical solutions they violated the "purity" of mathematics.

It was not until 1837 that the French mathematician PIERRE-LAURENT WANTZEL proved that a construction of the sought-after side length $x = \sqrt[3]{2} \cdot a \approx 1.26 \cdot a$ of the doubled altar using only compass and straight edge is not possible.

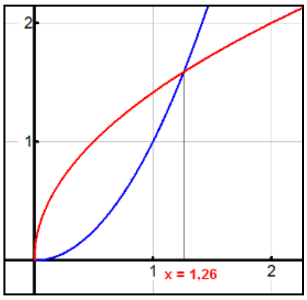
Hippocrates of Chios, who is also famous because of the *lunes* he discovered, was the first to realise that the problem can be solved if one can determine two mean proportions *x* and *y* to the edge length *a* of the altar, which fulfil the condition $a : x = x : y = y : 2a$, because inserting the condition $y = \frac{x^2}{a}$ into the equation $x : y = y : 2a$ results in $x^3 = 2a^3$, i.e. $x = \sqrt[3]{2} \cdot a \approx 1.26 \cdot a$.







Menaechmus, teacher of Alexander the Great, discovered that special curves occur when cones are cut, namely so-called parabolas and hyperbolas. He determined the solution with the help of two parabolas with the equations $y = \frac{1}{a} \cdot x^2$ and $x = \frac{1}{2a} \cdot y^2$, cf. right.

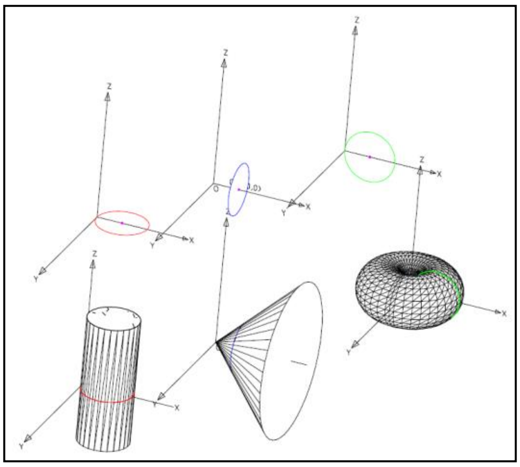


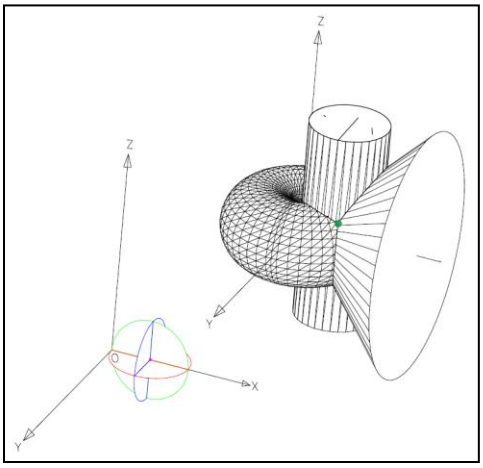
The solution of Archytas can (from today's point of view) perhaps be described most simply with the help of the methods of analytical geometry.

Three mutually perpendicular circles with radius *a* and centre (*a*, 0, 0) are considered, each of which lies parallel to the coordinate planes. A cone is drawn through the circle that is perpendicular to the *x*-axis, with its tip at the origin, and a cylinder is drawn through the circle in the *x*-*y* plane. The circle in the *x*-*z* plane is rotated around the *z*-axis so that a torus without a hole is created (so-called *horn torus*). Points on the surfaces of the bodies can be described by equations: $x^2 = y^2 + z^2$ (cone), $(x-a)^2 + y^2 = a^2$ (cylinder) and $(x^2 + y^2 + z^2)^2 = 4a^2 \cdot (x^2 + y^2)$ (torus).

These three surfaces have exactly one point in common whose *x*-coordinate is equal to $x = \sqrt[3]{2} \cdot a \approx 1.26 \cdot a$.

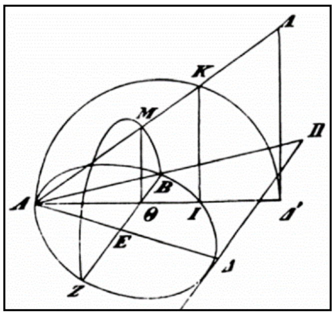
The following Wikimedia graphics by PMeg99 can help to imagine the spatial situation

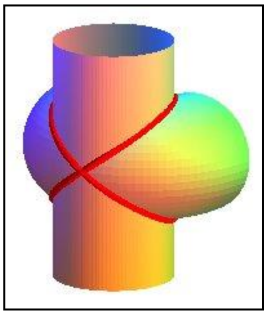


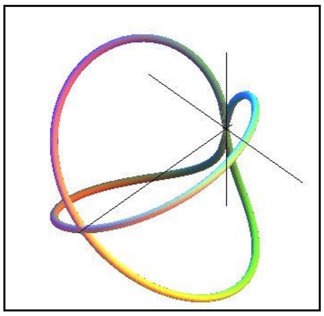


(https://it.wikipedia.org/wiki/Duplicazione_del_cubo)

The drawing on the right is taken from Moritz Cantor's *Lectures on the History of Mathematics*, Volume 1. There, a letter from Eratosthenes to Ptolemy is quoted, in which he explains how Archytas came to his discovery.







The intersection curve of torus and cylinder is called the *curve of Archytas*.

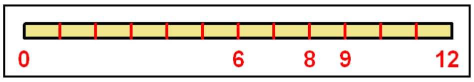
(Figures by Robert Ferréol: mathcurve.com)

Only fragments of the works of Archytas have survived, but much has become known through other authors of antiquity who have studied his works.

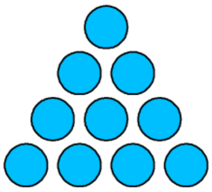
Among his philosophical-political views was the conviction that a predictable balanced distribution of property is a prerequisite for the social peace of a society. He fundamentally rejected punishing someone in anger, and rather refrained from punishment when no judgement without emotion was possible.

In his view, man arrives at scientific knowledge by progressing from the general to the particular. In doing so, he considered independent discovery to be more effective than learning from documented knowledge.

The Pythagoreans investigated regularities of musical sounds that were perceived as pleasant; in doing so, whole-number ratios played an important role:



If one halves the length of the strings of a monochord, then a tone is produced which is an octave higher than the fundamental tone (ratio 12:6 = 2:1). If the string length is shortened by a third, this corresponds to a tone that is a fifth above the fundamental (9:6 = 12:8 = 3:2), shortening the length by a quarter produces a fourth (12:9 = 8:6 = 4:3).



For the Pythagoreans, the fact that the numbers 1, 2, 3, 4 of the *Tetractys* appear here was confirmation of the existing world harmony.

Furthermore, if one divides the string length into 12 equal sections, then 9 is the arithmetic mean of 6 and 12 and 8 is the harmonic mean of 6 and 12 (the term *harmonic* comes from Archytas).

In his *Theory of means and proportions*, Archytas was able to prove the theorem that the geometric mean of two numbers cannot be rational if they are in the ratio n : (n+1); a subdivision of tone intervals is therefore only possible by forming arithmetic or harmonic means.

Euclid continued the music theory of Archytas in his writing *Sectio Canonis*. It can also be assumed that Euclid's doctrine of proportions (Book VII and VIII of the *Elements*) is essentially based on texts by Archytas.

Archytas held the view that sound is created by the collision of moving bodies and is transmitted as pressure through the air. Higher tones correspond to a faster movement of sound, and lower tones to a slower movement.

ARCHYTAS considered the universe to be infinitely large, i.e. not limited. For: *if it were limited and one were to move to the outermost edge – would it not then be paradoxical if one could not move one's own hand or a rod further outwards*?

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