NIELS HENRIK ABEL (August 5, 1802 – April 6, 1829)

by HEINZ KLAUS STRICK, Germany

When NIELS HENRIK ABEL was born, his homeland of Norway was on the front lines of a European war. Both NAPOLEON and Great Britain's allies had enforced a blockade against the country, which was thus cut off from other countries. Norway, which was only formally still a part of Denmark, was occupied by Swedish troops, and in 1814, was ceded to the king of Sweden. Eventually, Norway achieved independence.

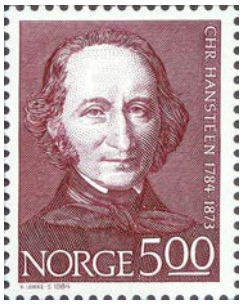
Although ABEL's father had a secure position as a Protestant minister, NIELS HENRIK ABEL grew up with his six brothers and sisters in difficult economic circumstances.

Initially, he was educated by his father; at age 13, he was permitted (together with his older brother) to attend the cathedral school in the capital, Christiania (today Oslo). At first, the transfer to this school failed to represent instructional improvement, since the formerly highly respected school had lost its entire faculty to the newly established university. Finally, the qualified mathematician BERNT HOLMBOË was engaged, who not only recognized the boy's unusual mathematical talent but was able to nurture it. He explained to him works of EULER and NEWTON.

It seemed to ABEL that those mathematicians' proofs of the generalized binomial theorem were not sufficiently rigorous, and he filled the gaps in their proofs with a precision that was unusual for that time. He also extended the statement of the theorem to an arbitrary real exponent.

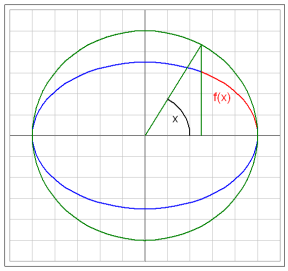
When ABEL was 18 years old, his alcohol-addicted father died; his unstable mother was unable to care adequately for her younger children. It seemed impossible on financial grounds that ABEL would be able to continue his education, and later study at university seemed out of the question.

HOLMBOË managed, however, to convince some friends to provide money for ABEL'S education. In his last year of secondary school, ABEL managed to solve, or so he believed, a hitherto unsolved famous problem of long standing, namely, that of expressing the solutions of a fifth-degree polynomial equation (solution methods for equations of degree 4 or less had been known since the Renaissance) in terms of the basic arithmetic operations and extraction of roots (such an equation is said to be "solvable in radicals"). But there was no one in Norway capable of following his argument, so it was proposed that he present his ideas to mathematicians at the University of Copenhagen. In reviewing his work, ABEL realized that he had made a significant error. It was this mistake that led him a few years later to understand why there can be no general algebraic procedure for solving an equation of degree 5.



At the age of 19, ABEL matriculated at the University of Christiania, where, however, there were no courses in higher mathematics. HOLMBOË convinced his former professor CHRISTOPHER HANSTEEN, who later became known for his research on the Earth's magnetic field, "to win this unusual talent for science".

HANSTEEN took a personal interest in ABEL, including financing a trip to Copenhagen so that he could discuss his ideas with the Danish mathematician FERDINAND DEGEN.



This gave ABEL the impetus to study the so-called elliptic integrals. For example, to determine the arc length of an ellipse (whence the name), certain integrals must be calculated: the function *f* defined by

$f(x) = a \cdot \int_0^x \sqrt{1-\varepsilon^2 \cos^2(t)}\,dt$ associates an angle *x* with the arc length *f*(*x*)

(*a* and *b* are the semiaxes of the ellipse, and ε is the numeric eccentricity).

Such integrals (including other related integrals) cannot be calculated in an elementary way; that is, one cannot express in terms of elementary functions a function whose derivative is the expression inside the integral sign. Therefore, the values of such integrals can be computed only numerically.

ABEL had the idea to investigate the associated inverse function, whereby a particular arc length of the ellipse is related to the associated angle, thereby creating the *theory of elliptic functions*.

He became convinced that to further his work, he would have to make personal contact with the greatest mathematicians of his time, namely CARL FRIEDRICH GAUSS in Göttingen and ADRIEN-MARIE LEGENDRE (the world expert on elliptic functions) and AUGUSTIN-LOUIS CAUCHY in Paris.

Over the following two years, he studied the French and German languages intensively and continued his work on mathematical problems. In 1824, he published his *Mémoire sur les équations algébriques ou on démontre l'impossibilité de la résolution de l'équation générale du cinquième degré*, though it appeared in an abridged form to save on the expense of publication, and he sent a copy to GAUSS, who put it aside unread.

Because of his limited contact with the scientific world, ABEL was unaware that PAOLO RUFFINI (1765–1822) had already, in 1799, published a paper expounding similar ideas, although his proof was marred by several gaps.

Finally, in 1825, he was granted the necessary funds for a journey. Carrying letters of introduction, he first visited the engineer August LEOPOLD CRELLE in Berlin, who at the time was preparing to launch a mathematical journal that would be independent of the universities, the *Journal für die reine und angewandte Mathematik* (Journal for Pure and Applied Mathematics).

The first issue of what came to be called "Crelle's journal" contained seven papers by ABEL, including the *Recherches sur les fonctions elliptiques* and *Beweis der Unmöglichkeit algebraische Gleichungen von höheren Graden als dem vierten allgemein aufzulösen* (proof of the impossibility of solving general algebraic equations of degree higher than the fourth).

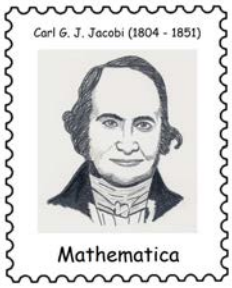
In the second of these papers, he gave a set of criteria for when an equation of higher degree could be solved in radicals. In the process of his research, he discovered the relationships that had to exist among the solutions (arrangements whose order could be reversed, whence commutative groups are called *abelian groups*).

Since he had been told that GAUSS was unapproachable, he did not travel to Göttingen, journeying instead to Paris via Freiburg, Dresden, Vienna, and Venice. However, he found there no suitable interlocutor. Eventually, he submitted to the *French Academy of Sciences* a paper on the generalization of an addition theorem for elliptic integrals (known today as *ABEL's theorem*); Legendre and CAUCHY were given the paper to review.

LEGENDRE's age rendered him unequal to the task, while CAUCHY recoiled at the very idea of having to read it at all. Indeed, he was so busy with his own publications that he simply put the paper aside and lost track of it. When the article was rediscovered after ABEL's death, he was awarded a posthumous prize from the Academy.

Abel was disappointed, for he knew that a positive judgment from CAUCHY would finally bring him the recognition that he sought. "CAUCHY is mad, and there is nothing that one can do about it, but he is the only one who knows how mathematics ought to be done."

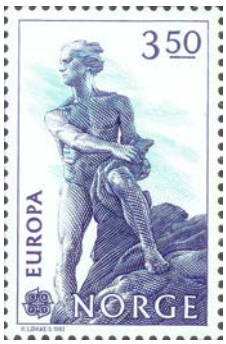
Discouraged, he travelled homeward. In Berlin, CRELLE tried to convince ABEL to stay. He offered to make him a coeditor of his journal; he also tried to obtain for him a position at the University of Berlin. But ABEL, in debt and worn out, wished to go home to Christiania, among other reasons to see his fiancée.



There, he was able only to obtain a position as a substitute teacher; then he filled in for HANSTEEN, who was on a major expedition to Siberia, at the university.

In poor health, he continued to work on elliptic functions: between him and CARL GUSTAV JACOB JACOBI, who independently had discovered similar approaches, there developed a fruitful yet debilitating rivalry. Through his publications, ABEL's reputation in the scientific world increased.

(drawings: © Andreas Strick)



Even GAUSS, who was also working intensively on elliptic functions but had published nothing on the subject, showed himself impressed with ABEL's contributions.

At the end of the year 1828, ABEL became severely ill; he had contracted tuberculosis in Paris. He died on April 6 of the following year. Two days later, CRELLE was informed by the University of Berlin that a teaching position would be offered to ABEL.

(to the left: Statue of NIELS HENRIK ABEL in Oslo)

First published 2009 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/niels-henrik-abel-1802-1829/993437

Translated by David Kramer

English version first published by the *European Mathematical Society* 2012

Here an important hint for philatelists who also like individual (not officially issued) stamps:

Enquiries at europablocks@web.de with the note: "Mathstamps"